Triangle constructions

Task: In Geogebra software construct in the given half plane triangles and discuss the number of solutions in connection to the positive real parameter t.

Exercise 1: Triangle ABC: $\mathrm{c} = 8\ \mathrm{cm}, |\sphericalangle \mathrm{ABC}| = 30°, \mathrm{b} = t\ \mathrm{cm}$

a) Solve for $t = 7$.
b) Solve with the positive real parameter t and hold a discussion.

Exercise 2 – for advanced students:

Triangle ABC: $\mathrm{c} = 4\ \mathrm{cm},\ \mathrm{v}_c = 6\ \mathrm{cm}, \mathrm{t}_a = t\ \mathrm{cm}$

a) Solve for $t = 7$.
b) Solve with the positive real parameter t and hold a discussion.

Procedure:

1. Copy the task into your school exercise book. Make a rough draft, write down the procedure of the construction for the target parameter t, construct and write the number of solutions in the given half plane.

2. In Geogebra software contruct the solution of the task with the circle *k* defined by the centre B and the point (with the variable radius). Choose the radius of the circle *k* so that the circle has two intersections with the straight line - as in exercise a).

3. V Geogebra software change the size of the circle radius and count the number of solutions and the individual shapes (acute-angled, obtuse-angled, right-angled triangle).

4. Write down into your school exercise book your observation in connection to the positive real parameter t, which shows the size of the radius circle *k*.

Methodological notes to solve the worksheet:

- you can add your rough drafts to solve the construction exercises on the board or assign the exercise for students in pair work.
- accompany the work in Geogebra software with the collective construction on the board or on the interactive whiteboard
- discuss together the number of solutions in connection to the size of the parameter t

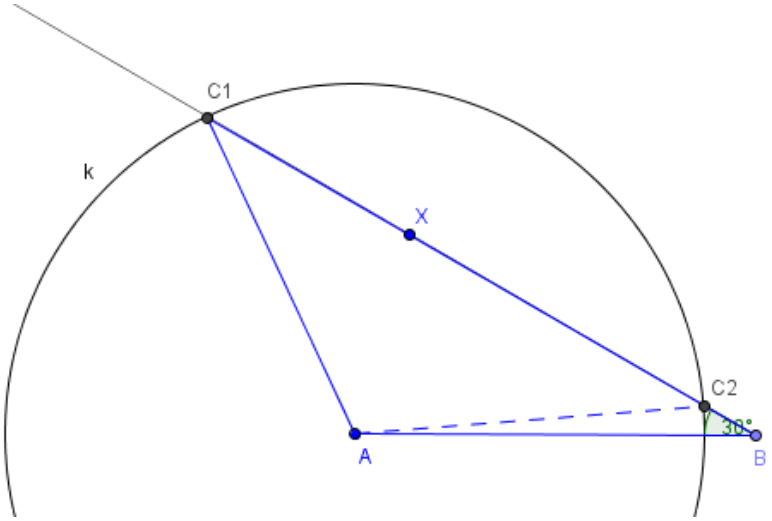
SOLUTION:

Exercise 1: Triangle ABC: $\mathrm{c} = 8\ \mathrm{cm}, |\sphericalangle \mathrm{ABC}| = 30°, \mathrm{b} = t\ \mathrm{cm}$

a) Solve for $t = 7$.
b) Solve with the positive real parameter t and hold a discussion.

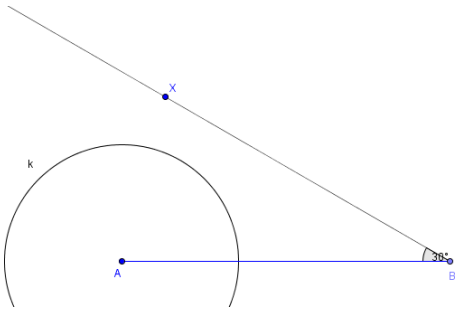
a) Construction notes:

- $AB;\ |AB| = 8\ cm$
- $\sphericalangle \mathrm{ABX};\ |\sphericalangle \mathrm{ABX}| = 30°$
- $k; k(A; 7\ cm)$
- $\mathrm{C}; \mathrm{C} \in \mathrm{k} \cap \rightarrow \mathrm{BX}$
- $\Delta\ ABC$

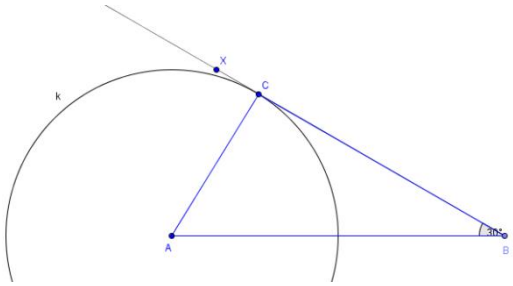


… two solutions $\Delta\ \mathrm{ABC}_1, \Delta\ \mathrm{ABC}_2$

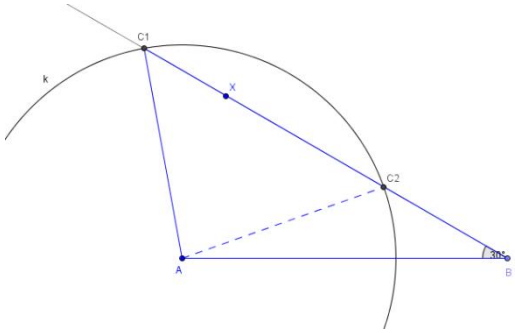
b) Discussion (number of solutions in the given half plane):



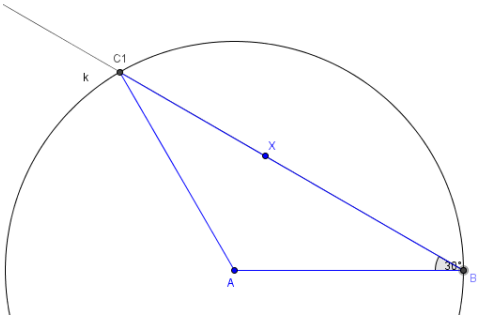
- $t \in (0;4) \implies 0\ solution$



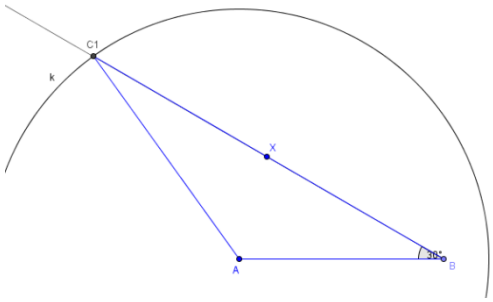
- $t \in \{4\} \implies 1\ solution$
  right-angled triangle



- $t \in (4;8) \implies 2\ solutions$
  at least one obtuse-angled triangle



- $t \in \{8\} \implies 1\ solution$
  obtuse-angled isosceles triangle
  (inner angles 120°, 30°, 30°, $|AB| = |AC|$)



- $t \in (8;\infty) \implies 1\ solution$
  obtuse-angled triangle

Solution:

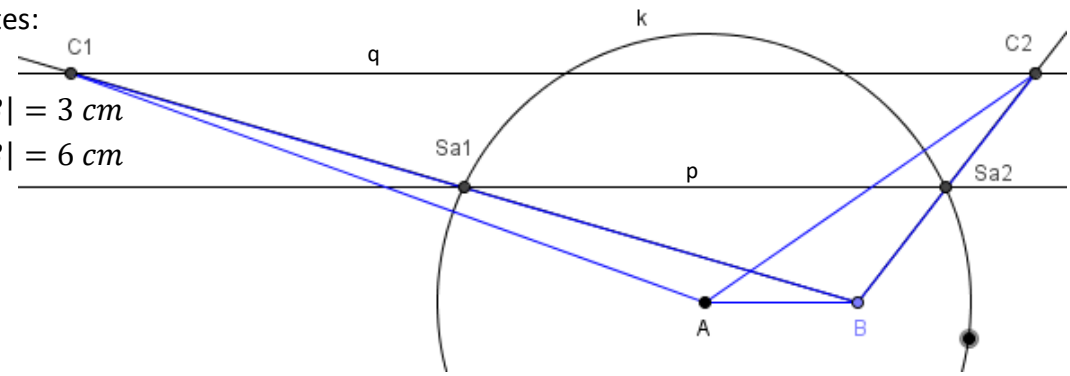
Exercise 2: Triangle ABC: $c = 4\ \text{cm},\ v_c = 6\ \text{cm}, t_a = t\ \text{cm}$

a) Solve for $t = 7$.

b) Solve with the positive real parameter t and hold a discussion.

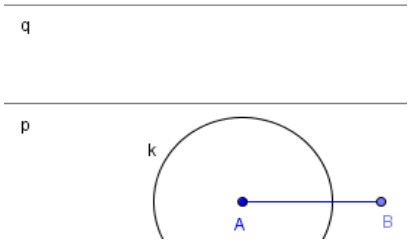
a) Construction notes:

- $AB;\ |AB| = 4\ cm$
- $p; p \parallel AB \wedge |p; AB| = 3\ cm$
- $q; q \parallel AB \wedge |q; AB| = 6\ cm$
- $k; k(A; 7\ cm)$
- $S_a;\ S_a \in k \cap p$
- $C; C \in q \cap \rightarrow BS_a$
- $\Delta\ ABC$

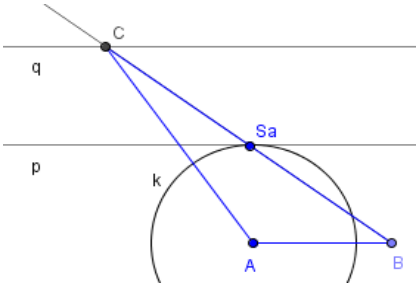


… two solutions $\Delta\ ABC_1, \Delta\ ABC_2$

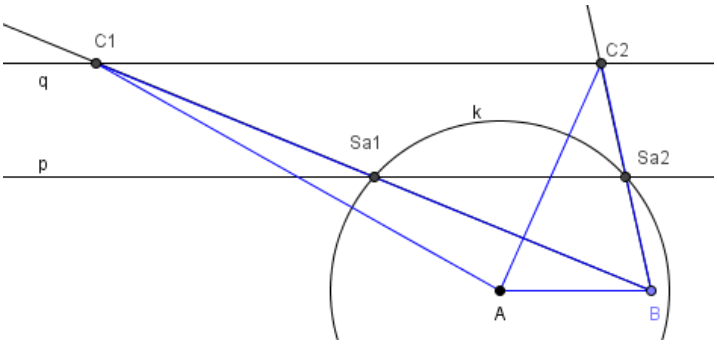
b) Discussion (number of solutions in the given half plane):



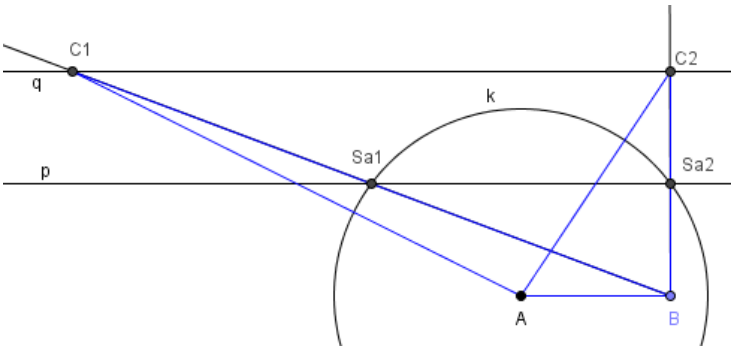
- $t \in (0;3) \implies 0\ solution$



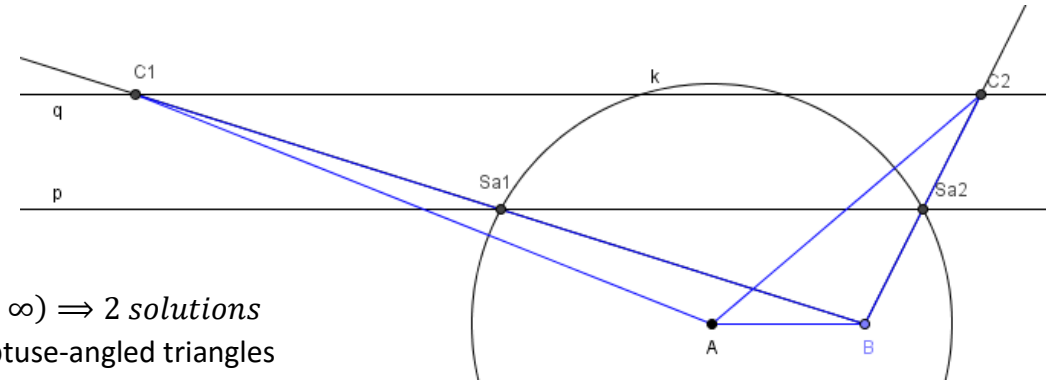
- $t \in \{3\} \implies 1\ solution$
  obtuse-angled triangle



- $t \in (3;5) \implies 2\ solutions$
  at least one obtuse-angled triangle



- $t \in \{5\} \implies 2\ solutions$
  obtuse-angled and right-angled triangle



- $t \in (5;\infty) \implies 2\ solutions$
  two obtuse-angled triangles